I B.Tech II Semester $\begin{gathered}\text { Supplementary Examinations, Feb/Mar } 2014 \\ \text { MATHEMATICS- II }\end{gathered}$
( Common to Civil Engineering, Electrical \& Electronics Engineering, Mechanical Engineering, Electronics \& Communication Engineering, Computer Science \& Engineering, Chemical Engineering, Electronics \& Instrumentation Engineering, Bio-Medical Engineering, Information Technology, Electronics \& Computer Engineering, Aeronautical
Engineering, Bio-Technology, Automobile Engineering, Mining and Petroliem Technology)
Time: 3 hours
Answer any FIVE Questions
All Questions carry equal marks

1. (a) Find $\mathrm{L}\left(\mathrm{t} \mathrm{e}^{2 t} \sin 3 \mathrm{t}\right)$
(b) Find $\mathrm{L}\left(\frac{\sin t}{t}\right)$
2. (a) Find $L^{-1}\left[\frac{s-3}{s^{2}-10 s+29}\right]$.
(b) Find $L^{-1}\left[\log \left(\frac{s^{2}+4}{s^{2}+9}\right)\right]$.
3. Find the fouries series for the function

$$
\begin{align*}
& f(x)=-\pi,-\pi<x<0 \\
& \quad=\pi, 0<x<\pi \text { and hence deduce the series } \frac{1}{1^{2}}+\frac{1}{3^{2}}+\frac{1}{5^{2}}+\ldots . .=\frac{\pi^{2}}{8} \tag{15}
\end{align*}
$$

4. (a) Find the fourier transform of $\mathrm{f}(\mathrm{x})$ defined by $\mathrm{f}(\mathrm{x})=\mathrm{e}^{i a x}, \alpha<\mathrm{x}<\beta, \mathrm{f}(\mathrm{x})=0, \mathrm{x}<$ $\alpha$ and $\mathrm{x}>\beta$
(b) Find the fourier transform of $\mathrm{f}(\mathrm{x})$ defined by $\mathrm{f}(\mathrm{x})=\mathrm{e}^{-x 2 / 2},-8<\mathrm{x}<8$. $[8+7]$
5. (a) Form the Partial Differential Equation by eliminating arbitrary functions from $z=y f(x)+x g(y)$
(b) Solve $\mathrm{p}+\mathrm{q}=1$
6. An insulated rod of length $l$ has its ends A and B maintained at $0^{\circ} \mathrm{C}$ and $100^{\circ} \mathrm{C}$ respectively until steady state conditions prevail. If the ends A and B are changed to $40^{\circ} \mathrm{C}$ and $60^{\circ} \mathrm{C}$ and maintained at these values, find the transient distribution of the rod.
7. (a) Find $Z^{-1}\left(\frac{2 z^{2}+3 z}{(z+2)(z-4)}\right)$
(b) Find the inverse Z-transform of $\frac{5 z}{(2-z)(3 z-1)}$.
8. (a) Prove that $\beta(m, n)=a^{m} b^{n} \int_{0}^{\infty} \frac{x^{m-1}}{(a x+b)^{m+n}} d x$, where a,b $>0$.
(b) Express $\int_{0}^{1} \frac{1}{\left(1-x^{3}\right)} 1 / 3$ in terms of gamma functions.

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Time: 3 hours
Max Marks: 75
Answer any FIVE Questions
All Questions carry equal marks

1. (a) Find the Laplace transform of $\mathrm{e}^{5 t}+\mathrm{e}^{-2 t}-\sin 5 \mathrm{t}+\cos 4 \mathrm{t}-\sinh 2 \mathrm{t}+5 \cosh 3 \mathrm{t}$ $+t^{5}-9$
(b) Find the Laplace transform of $\mathrm{t}^{2} \sin 2 \mathrm{t}$
2. (a) Find inverse Laplace transform of $\frac{4 s+5}{(s-1)^{2}(s-2)}$
(b) Find inverse Laplace transform of $\frac{1}{s(s+3)^{3}}$
3. Obtain the half range cosine and sine series for $f(x)=x$ in $o<x<L$
4. Find the fourier sine transform of $\mathrm{f}(\mathrm{x})=\frac{1}{x\left(x^{2}+a^{2}\right.}$
5. (a) Solve $p-q=z-y$
(b) Solve ( $x-a$ ) $p+(y-b) q=z-c$
6. (a) Solve $3 \mathrm{u}_{x}+2 \mathrm{u}_{y}=0$ and $\mathrm{u}(\mathrm{x}, \mathrm{o})=4 \mathrm{e}^{-x}$ by the Method of Separation of Variables.
(b) Solve $\frac{d^{2} z}{d x^{2}}-2 \frac{d z}{d x}+\frac{d z}{d y}=0$ by the Method of Separation of Variables. [8+7]
7. (a) Find $Z^{-1}\left(\frac{3 z^{2}+z}{(5 z-1)(5 z+2)}\right)$.
(b) Find $Z^{-1}\left(\frac{z^{2}-3 z}{(z+2)(z-5)}\right)$
8. (a) Prove that $\beta(m, n)=\frac{\Gamma(m) \Gamma(n)}{\Gamma(m+n)}$.
(b) Express $\int_{0}^{\pi / 2} \sqrt{\tan \theta} d \theta$ in terms of gamma functions.

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Engineering, Bio-Technology, Automobile Engineering, Mining and Petroliem Technology)
Time: 3 hours
Max Marks: 75
Answer any FIVE Questions
All Questions carry equal marks

1. (a) Find the Laplace transform of $t \sin 2 t \cos 2 t$
(b) Find $L\left(t^{2} \cos 2 t\right)$
2. (a) Find $L^{-1}\left[e^{-2 s} /\left(s^{2}+4 s+5\right)\right]$.
(b) Find $L^{-1}\left[\log \left(\frac{s+1}{s-1}\right)\right]$.
3. Find the fourier series for $f(x)=2 L x-x^{2}$ in $0<x<2 L$ and hence deduce $1-1 / 2^{2}+1 / 3^{2}-$ $1 / 4^{2}+\ldots=\pi^{2} / 12$
4. Find the fourier sine transform of $f(x)=\left\{\begin{array}{cl}1-x^{2}, & |z|<1 \\ 0 & |z|>1\end{array}\right.$
5. (a) Form the Partial Differential Equation by eliminating arbitrary function from $f\left(x^{2}+y^{2}, z-x y\right)=0$
(b) Solve $\mathrm{pq}=\mathrm{xy}$
6. A tightly stretched string of length $l$ with fixed end points is initially at rest in its equilibrium position and each of its points is given a velocity $\mathrm{v}(\mathrm{x})$ such that $v(x)=\left\{\begin{array}{ll}c x, & 0 \leq x \leq \frac{l}{2} \\ c(l-x), & \frac{l}{2} \leq x \leq l\end{array}\right.$ find the displacement of any point on the string at any time ' t '.
7. (a) Find the Z-transforms of (i) $e^{-a n} \sin n \theta$
(ii) $3 n^{2}+10 \cos \left(\frac{\mathrm{n} \pi}{2}\right)+a^{n+2}$.
(b) Find the Z-transforms of (i) $(\mathrm{n}-1)^{2}$
(ii) $5 \mathrm{e}^{-\mathrm{an}} \sin \left(\frac{\mathrm{n} \pi}{4}\right)-3 a^{4}$. $[8+7]$
8. (a) Show that $\int_{0}^{\infty} e^{-x^{2}} d x=\frac{\sqrt{\pi}}{2}$
(b) Prove that $\beta(m+1, n)+\beta(m, n+1)=\beta(m, n)$

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Time: 3 hours
Max Marks: 75
Answer any FIVE Questions
All Questions carry equal marks

1. (a) Find the Laplace transform of $\cos 3 t \sin 5 t$
(b) Find the Laplace transform of $\frac{1-e^{-t}}{t}$
2. (a) Find $L^{-1}\left[(2 s+3) /\left(s^{3}-6 s^{2}+11 s-6\right)\right]$.
(b) Find $L^{-1}\left\{\cot ^{-1} s\right\}$.
3. $f(x)=k x, \quad$ for $0<x<\frac{\pi}{2}$

$$
\begin{equation*}
k(\pi-x), \quad \text { for } \frac{\pi}{2}<x<\pi \text { find the half range sine series } \tag{15}
\end{equation*}
$$

4. (a) Find the inverse fourier cosine transform of $f(x)$ of $\mathrm{F}_{c}(\mathrm{p})=1 / 2 \mathrm{a}(\mathrm{a}-\mathrm{p} / 2)$ when $\mathrm{p}<2 \mathrm{a}, \quad \mathrm{F}_{c}(\mathrm{p})=0$ when $\mathrm{p}=2 \mathrm{a}$
(b) Find the fourier cosine transform of $\mathrm{f}(\mathrm{x})=\mathrm{e}^{-a x} \cos \mathrm{ax}$
5. (a) Form the Partial Differential Equation by eliminating arbitrary function from $f(x+z, y+z)=0$.
(b) Solve $\mathrm{x}\left(\mathrm{y}^{2}-\mathrm{z}^{2}\right) \mathrm{p}+\mathrm{y}\left(\mathrm{z}^{2}-\mathrm{x}^{2}\right) \mathrm{q}=\mathrm{z}\left(\mathrm{x}^{2}-\mathrm{y}^{2}\right)$
6. Solve $\frac{\partial^{2} u}{\partial t^{2}}=\alpha^{2} \frac{\partial^{2} u}{\partial x^{2}},-\alpha<x<\alpha, t \geq 0$ with conditions $\mathrm{u}(\mathrm{x}, 0)=\mathrm{f}(\mathrm{x})$ and $\left(\frac{\partial u}{\partial t}\right)_{(x, o)}=g(x)$ assuming $\mathrm{u}, \frac{\partial u}{\partial t} \rightarrow 0$ as $x \rightarrow \alpha$.
7. (a) Find the inverse Z-transform of $\frac{z}{(z-1)(z-2)}$
(b) Determine $u_{2}$ where $U(z)=\frac{2 z^{2}+3 z+4}{(z-3)^{3}}, \quad|z|>3$
8. (a) Show that $\int_{-1}^{1}(1+x)^{m-1}(1-x)^{n-1} d x=2^{m+n-1} \beta(m, n)$
(b) Show that $\int_{0}^{\infty} \frac{\mathrm{x}^{\mathrm{n}}}{\mathrm{n}^{\mathrm{x}}} d x=\frac{\Gamma(n+1)}{(\log n)^{n+1}}, \mathrm{n}>1$
